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The quarkonia-glueball structure of η, η' and $\eta(1410)$ revisited

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Abstract. The quarkonia-glueball structure of η, η' and $\eta(1440)$ is phenomenologically deduced from the updated world average data including two-photon decays of pseudoscalar mesons, radiative decays between pseudoscalar and vector mesons, decays of J/ψ into a vector and a pseudoscalar meson, and radiative decays of J/ψ. Corrections due to SU(3) flavor symmetry breaking, non-ideal ω–ϕ mixing, double OZI and electromagnetic amplitudes of J/ψ decays into a vector and a pseudoscalar meson, and three-gluon-annihilation amplitudes of radiative decays of J/ψ are included in our analysis. Some predictions on the decays involving $\eta(1410)$ are presented, which can be tested at Beijing Electron Positron Collider/Beijing Spectrometer (BEPC/BES) with 50 million J/ψ events.

1 Introduction

The $1\ ^1S_0$ meson nonet is well established, the isotriplet $\pi(1300)$ and the isodoublet $K(1460)$ of the $2\ ^1S_0$ meson nonet have been established [1] and the $\eta(1295)$ is generally accepted as the first radial excitation of η [2,3]. In addition, the pseudoscalar glueball candidate $\eta(1440)$ [1] has been resolved into two states [4–6]: $\eta(1490)$ and $\eta(1410)$ (η'' represents $\eta(1410)$ below). The former can be interpreted as the first radial excitation of η' [2,3,7], and the latter seems a spurious state, which is argued to be mainly a glueball, possibly mixed with $q\bar{q}$ states [7].

In general, states with the same isospin–spin–parity IJ^{PC} and additive quantum numbers can mix. The masses of $\pi(1300)$ and $\eta(1295)$ are almost equal, which implies that $\eta(1295)$ and its isoscalar partner are almost ideally mixing [8]. Therefore, it is expected that the possibility of the isoscalar states of the $2\ ^1S_0$ nonet mixing with those of the $1\ ^1S_0$ nonet or the pseudoscalar glueball can be ignored; then one naturally focuses on the mixing of η, η' and η''.

In the past thirty years, the mixing of η, η' and η'' has been discussed many times, and the quarkonia-glueball structure of η, η' and η'' has been deduced from the world average data on some of the decays such as $P\ (V) \to \gamma V\ (P)$, $P \to \gamma\gamma$, $J/\psi \to PV$ and $J/\psi \to \gamma P$ (P and V denote the light pseudoscalar and vector mesons, respectively) involving η and η' [9–12]. More recently, Carvalho et al. [13] deduced the quarkonia-glueball content of η, η' and η'' from part of the data on $P\ (V) \to \gamma V\ (P)$ and $J/\psi \to PV$. In the analysis of $J/\psi \to PV$ in [13], single OZI (SOZI) amplitude was considered while corrections due to SU(3) flavor breaking, DOZI and electromagnetic amplitudes were neglected. The importance of these corrections has been emphasized in [14–18]. All corrections mentioned above to $J/\psi \to PV$ and corrections originating from three-gluon-annihilation amplitudes to $J/\psi \to \gamma P$ were taken into account in [12]. In the analysis of [12], there exists the explicit assumption that $r_0'' = \frac{r_0'}{r_0}$, where r_0'' denotes the coupling strength of Fig. 2(ii) relative to that of Fig. 2(i), and $\frac{r_0'}{r_0}$ denotes the coupling strength of Fig. 1(iv) relative to that of Fig. 1(iii). This assumption may be valid in the viewpoint of the perturbative theory, however, the processes as shown in Fig. 1(iii), (iv) and Fig. 2(i), (ii), in principle, should be non-perturbative processes; therefore, there might not be convincing reasons to expect that r_0'', r_0' and r_0 should behave as $r_0'' = \frac{r_0'}{r_0}$.

The world average data on the $J/\psi \to PV$ and $J/\psi \to \gamma P$ nearly remains unchanged in the recent editions of the PDG compilations, however, part of the data on $V\ (P) \to P\ (V)\gamma$ are apparently updated, especially the data on $\phi \to \gamma\eta'$, as averaged by the new edition of the PDG compilation [19], which are crucial to the $s\bar{s}$ content of η', are remarkably different from the old data [20].

The purpose of this work is to re-deduce the quarkonia-glueball content of η, η' and η'' from the updated and complete set of data on $P\ (V) \to \gamma V\ (P)$, $P \to \gamma\gamma$, $J/\psi \to PV$ and $J/\psi \to \gamma P$. We will follow quite closely the similar analysis in [12] except that the argument given in [12] that $r_0'' = \frac{r_0'}{r_0}$ is not adopted in our present work, and that non-ideal ω–ϕ mixing corrections will now be taken into account. Our present analysis can be considered as an improvement of the previous work mentioned above.

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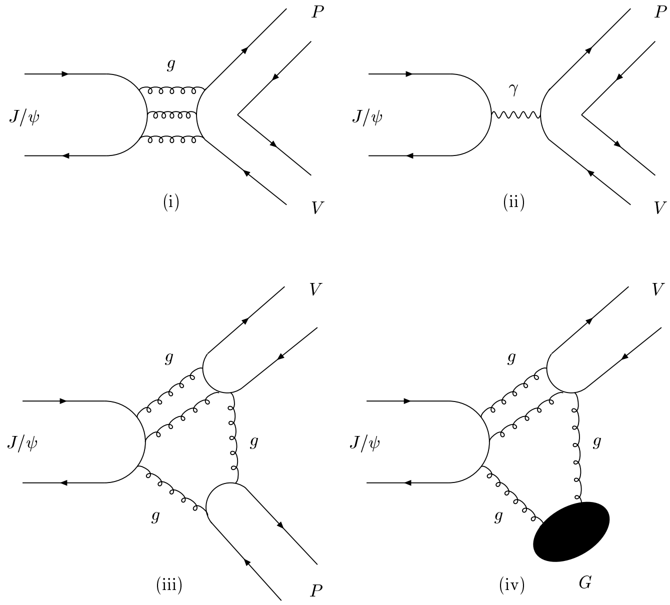

Fig. 1(i)–(iv). Diagrams contributing to $J/\psi \to PV$: **(i)** single-OZI-suppression diagram, **(ii)** electromagnetic decay diagram, **(iii)** double-OZI-suppression diagram connected to $q\bar{q}$ states and **(iv)** double-OZI-suppression diagram connected to a $q\bar{q}$ state and a glueball state

2 Mixing scheme and decays

Based on three Euler angles θ_1, θ_2 and θ_3, the mixing of η, η' and η'' can be described as [21]

$$\begin{pmatrix} \eta \\ \eta' \\ \eta'' \end{pmatrix} = \begin{pmatrix} a_8 & a_1 & a_g \\ b_8 & b_1 & b_g \\ c_8 & c_1 & c_g \end{pmatrix} \begin{pmatrix} |8\rangle \\ |1\rangle \\ |G\rangle \end{pmatrix} = \begin{pmatrix} x_\eta & y_\eta & z_\eta \\ x_{\eta'} & y_{\eta'} & z_{\eta'} \\ x_{\eta''} & y_{\eta''} & z_{\eta''} \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}, \tag{1}$$

with

$$\begin{pmatrix} a_8 & a_1 & a_g \\ b_8 & b_1 & b_g \\ c_8 & c_1 & c_g \end{pmatrix} = \begin{pmatrix} c_1c_2c_3 - s_1s_3 & -c_1c_2s_3 - s_1c_3 & c_1s_2 \\ s_1c_2c_3 + c_1s_3 & -s_1c_2s_3 + c_1c_3 & s_1s_2 \\ -s_2c_3 & s_2s_3 & c_2 \end{pmatrix}, \tag{2}$$

$$\begin{pmatrix} x_\eta & y_\eta & z_\eta \\ x_{\eta'} & y_{\eta'} & z_{\eta'} \\ x_{\eta''} & y_{\eta''} & z_{\eta''} \end{pmatrix} = \begin{pmatrix} a_8 & a_1 & a_g \\ b_8 & b_1 & b_g \\ c_8 & c_1 & c_g \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} & 0 \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3}$$

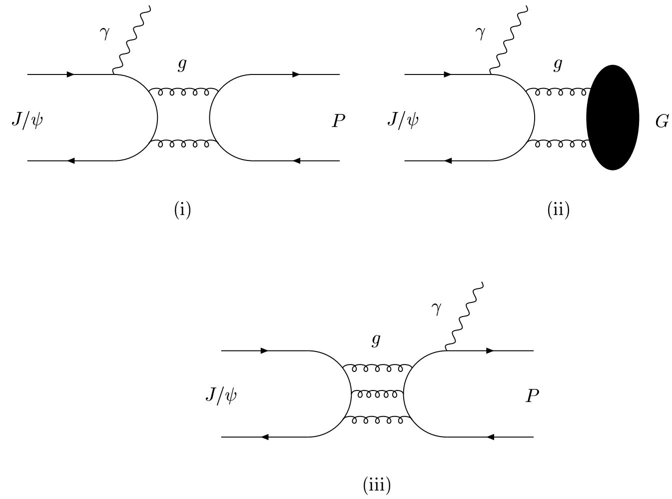

Fig. 2(i)–(iii). Diagrams contributing to $J/\psi \to \gamma P$: **(i)** two-gluon-annihilation diagram connected to a $q\bar{q}$ state, **(ii)** two-gluon-annihilation diagram connected to a glueball state and **(iii)** three-gluon-annihilation diagram

where $|8\rangle = |u\bar{u} + d\bar{d} - 2s\bar{s}\rangle/\sqrt{6}$, $|1\rangle = |u\bar{u} + d\bar{d} + s\bar{s}\rangle/\sqrt{3}$, $|N\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$, $|S\rangle = |s\bar{s}\rangle$, $|G\rangle = |gg\rangle$; c_1 $(c_2,\ c_3)\equiv \cos\theta_1$ $(\cos\theta_2,\ \cos\theta_3)$, s_1 $(s_2,\ s_3)\equiv \sin\theta_1$ $(\sin\theta_2,\ \sin\theta_3)$, and $-180^\circ \le \theta_1 \le 180^\circ$, $0^\circ \le \theta_2 \le 180^\circ$, $-180^\circ \le \theta_3 \le 180^\circ$. One advantage of this mixing model is that there are only three unknown parameters with definite changing ranges.

If we consider SU(3) flavor symmetry breaking corrections arising from the constituent quark mass differences, the effective lagrangian for $P \to \gamma\gamma$ can be written as [12, 22]

$$\mathcal{L}_{P\gamma\gamma} = g_1\ \mathbf{Tr}(S_e QP), \tag{4}$$

where g_1 is the coupling strength of $P \to \gamma\gamma$,

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \sum\limits_i \frac{x_i}{\sqrt{2}} i & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \sum\limits_i \frac{x_i}{\sqrt{2}} i & K^0 \\ K^- & \overline{K}^0 & \sum\limits_i y_i i \end{pmatrix},$$

which describes the pseudoscalar meson nonet, here and below, $i = \eta$, η' and η'', Q is the quark-charge matrix (for u, d and s quarks, $Q = \mathrm{diag}\{2/3, -1/3, -1/3\}$), $S_e = \mathrm{diag}\{2/3, -1/3, -R/3\}$ is the matrix of the spurious field corresponding to the electromagnetic interaction, $R \equiv m_u/m_s$ represents the SU(3) violation factor, m_u and m_s are respectively the masses of the constituent quarks u and s. Here we assume that the constituent quarks u and d have the same mass. From (4), one can extract the theoretical decay rate as

$$\frac{\Gamma(i \to \gamma\gamma)}{\Gamma(\pi \to \gamma\gamma)} = \frac{1}{9}\left(\frac{m_i}{m_\pi}\right)^3 (5x_i + \sqrt{2}Ry_i)^2, \tag{5}$$

where m_i and m_π are the masses of i and π, respectively.

Similarly, if the effects of SU(3) flavor symmetry breaking are taken into account, the phenomenological lagrangian describing the decay processes $V \to P\gamma$ and $P \to V\gamma$ can be written as [12,22]

$$\mathcal{L}_{VP\gamma} = g_2 \ \mathbf{Tr}(S_e\{V, P\}), \tag{6}$$

where g_2 is the coupling strength of $V \to P\gamma$ or $P \to V\gamma$,

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{x_\omega\omega + x_\phi\phi}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{x_\omega\omega + x_\phi\phi}{\sqrt{2}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & y_\omega\omega + y_\phi\phi \end{pmatrix},$$

which describes the vector meson nonet. Here, we take into account the effects of non-ideal ω–ϕ mixing by writing the physical states ω and ϕ as

$$\omega = x_\omega \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} + y_\omega s\bar{s}, \ \phi = x_\phi \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} + y_\phi s\bar{s}.$$

From (6), one can obtain the following theoretical decay rates:

$$\begin{aligned}
\frac{\Gamma(\rho \to \eta\gamma)}{\Gamma(\omega \to \pi\gamma)} &= \left[\frac{(m_\rho^2 - m_\eta^2)m_\omega}{(m_\omega^2 - m_\pi^2)m_\rho}\right]^3 \frac{x_\eta^2}{x_\omega^2}, \\
\frac{\Gamma(\omega \to \eta\gamma)}{\Gamma(\omega \to \pi\gamma)} &= \frac{1}{9}\left[\frac{m_\omega^2 - m_\eta^2}{m_\omega^2 - m_\pi^2}\right]^3 \frac{(x_\omega x_\eta - 2y_\omega R y_\eta)^2}{x_\omega^2}, \\
\frac{\Gamma(\phi \to \eta\gamma)}{\Gamma(\omega \to \pi\gamma)} &= \frac{1}{9}\left[\frac{(m_\phi^2 - m_\eta^2)m_\omega}{(m_\omega^2 - m_\pi^2)m_\phi}\right]^3 \frac{(x_\phi x_\eta - 2y_\phi R y_\eta)^2}{x_\omega^2}, \\
\frac{\Gamma(\phi \to \eta'\gamma)}{\Gamma(\omega \to \pi\gamma)} &= \frac{1}{9}\left[\frac{(m_\phi^2 - m_{\eta'}^2)m_\omega}{(m_\omega^2 - m_\pi^2)m_\phi}\right]^3 \\
&\quad \times \frac{(x_\phi x_{\eta'} - 2y_\phi R y_{\eta'})^2}{x_\omega^2}, \\
\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi\gamma)} &= 3\left[\frac{(m_{\eta'}^2 - m_\rho^2)m_\omega}{(m_\omega^2 - m_\pi^2)m_{\eta'}}\right]^3 \frac{x_{\eta'}^2}{x_\omega^2}, \\
\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \pi\gamma)} &= \frac{1}{3}\left[\frac{(m_{\eta'}^2 - m_\omega^2)m_\omega}{(m_\omega^2 - m_\pi^2)m_{\eta'}}\right]^3 \\
&\quad \times \frac{(x_\omega x_{\eta'} - 2y_\omega R y_{\eta'})^2}{x_\omega^2},
\end{aligned} \tag{7}$$

where m_ρ, m_ω and m_ϕ are the masses of ρ, ω and ϕ, respectively.

Considering the effects of SU(3) flavor breaking, DOZI and electromagnetic amplitudes, the effective lagrangian for $J/\psi \to PV$ can be expressed by [12,14,23],

$$\begin{aligned}
\mathcal{L}_{J/\psi PV} &= \frac{g_0}{2} \ \mathbf{Tr}(P\{V, S_v\}) + \frac{e_0}{2} \ \mathbf{Tr}(P\{V, S_e\}) \\
&\quad + r_0 g_0 \ \mathbf{Tr}(PS_p)\mathbf{Tr}(VS_v) + r_0' g_0 \ G \ \mathbf{Tr}(VS_v),
\end{aligned} \tag{8}$$

where g_0, e_0 (with phase θ_e relative to g_0), $r_0 g_0$ and $r_0' g_0$ are the coupling strengths of the processes shown in Fig. 1(i), (ii), (iii) and (iv), respectively, S_v and S_p can be written as

$$S_v = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - s_v \end{pmatrix}, \ S_p = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - s_p \end{pmatrix},$$

where the spurious fields S_v and S_p correspond to the three-gluon-annihilation and the two-gluon-annihilation interactions, respectively, S_v (S_p) is the almost SU(3) singlet with small violation s_v (s_p) which represents the SU(3) violation due to the difference between the strange and non-strange quarks.

Considering three-gluon-annihilation amplitudes corrections, the effective lagrangian for $J/\psi \to \gamma P$ can be expressed by [12]

$$\mathcal{L}_{J/\psi\gamma P} = d_0 \ \mathbf{Tr}(S_p P) + r_0'' d_0 G + \frac{f_0}{2} \ \mathbf{Tr}(S_v\{P, S_e\}), \tag{9}$$

where d_0, $r_0'' d_0$ and f_0 are the effective coupling strengths of the processes shown in Fig. 2(i), (ii) and (iii), respectively. Here, we take r_0'' as a free parameter rather than $r_0'' = r_0'/r_0$ used in [12].

If we set $\sqrt{3}g = g_0$, $6e = e_0$, $\sqrt{3}r = r_0$, $r' = r_0'$, $\sqrt{6}d = d_0$, $6\sqrt{6}f = f_0$ and $r'' = r_0''$, from (8) and (9), one can obtain the reduced amplitudes for the decays of $J/\psi \to PV$ (γ) as listed in (10). The various branching ratios, $\mathrm{Br}(J/\psi \to PV \ (\gamma))$ can be simply related to the corresponding amplitudes in (10) by $\mathrm{Br}(J/\psi \to PV \ (\gamma)) = C_{PV(\gamma)}|\mathrm{Am}(PV \ (\gamma))|^2|\mathbf{p}|^3$, where $\mathrm{Br}(J/\psi \to PV \ (\gamma))$ is given in 10^{-3} if the final meson momentum $\mathbf{p}$ in the center of mass is given in GeV, and $C_{PV(\gamma)}$ is a weighting factor arising from the sum over the various charge combinations, namely 3 for $\rho\pi$, 2 for $K^+\overline{K}^{*-}$ + c.c., 2 for $K^0\overline{K}^{*0}$ + c.c. and 1 for the other channels listed in (10). We have

$$\begin{aligned}
&\mathrm{Am}(\rho\pi) = g + e, \\
&\mathrm{Am}(K^+\overline{K}^{*-} + \text{c.c.}) = g\left(1 - \frac{s_v}{2}\right) + e(2 - R), \\
&\mathrm{Am}(K^0\overline{K}^{*0} + \text{c.c.}) = g\left(1 - \frac{s_v}{2}\right) - e(1 + R), \\
&\mathrm{Am}(\omega\eta) \\
&= x_\omega\Big\{(g + e)x_\eta + \sqrt{2}rg\left[\sqrt{2}x_\eta + (1 - s_p)y_\eta\right] \\
&\quad + \sqrt{2}r'gz_\eta\Big\} \\
&+ y_\omega\Big\{\left[g(1 - s_v) - 2eR\right]y_\eta \\
&\quad + rg(1 - s_v)\left[\sqrt{2}x_\eta + (1 - s_p)y_\eta\right]\Big\} \\
&+ y_\omega \ r'g(1 - s_v)z_\eta, \\
&\mathrm{Am}(\omega\eta') \\
&= x_\omega\Big\{(g + e)x_{\eta'} + \sqrt{2}rg\left[\sqrt{2}x_{\eta'} + (1 - s_p)y_{\eta'}\right] \\
&\quad + \sqrt{2}r'gz_{\eta'}\Big\} \\
&+ y_\omega\Big\{\left[g(1 - s_v) - 2eR\right]y_{\eta'} \\
&\quad + rg(1 - s_v)\left[\sqrt{2}x_{\eta'} + (1 - s_p)y_{\eta'}\right]\Big\} \\
&+ y_\omega \ r'g(1 - s_v)z_{\eta'},
\end{aligned}$$

$$
\begin{aligned}
&\mathrm{Am}(\phi\eta)\\
&= x_\phi\Big\{(g+e)x_\eta + \sqrt{2}rg\left[\sqrt{2}x_\eta + (1-s_p)y_\eta\right]\\
&\quad +\sqrt{2}r'gz_\eta\Big\}\\
&+ y_\phi\Big\{\left[g(1-s_v) - 2eR\right]y_\eta\\
&\quad +rg(1-s_v)\left[\sqrt{2}x_\eta + (1-s_p)y_\eta\right]\Big\}\\
&+ y_\phi\, r'g(1-s_v)z_\eta,\\
&\mathrm{Am}(\phi\eta')\\
&= x_\phi\Big\{(g+e)x_{\eta'}\\
&\quad +\sqrt{2}rg\left[\sqrt{2}x_{\eta'} + (1-s_p)y_{\eta'}\right] + \sqrt{2}r'gz_{\eta'}\Big\}\\
&+ y_\phi\Big\{\left[g(1-s_v) - 2eR\right]y_{\eta'}\\
&\quad +rg(1-s_v)\left[\sqrt{2}x_{\eta'} + (1-s_p)y_{\eta'}\right]\Big\}\\
&+ y_\phi\, r'g(1-s_v)z_{\eta'},\\
&\mathrm{Am}(\rho\eta) = 3ex_\eta,\\
&\mathrm{Am}(\rho\eta') = 3ex_{\eta'},\\
&\mathrm{Am}(\omega\pi) = 3x_\omega e,\\
&\mathrm{Am}(\phi\pi) = 3x_\phi e,\\
&\mathrm{Am}(\gamma\eta)\\
&= 2(d+f)x_\eta + \sqrt{2}\left[(1-s_p)d - 2fR(1-s_v)\right]y_\eta\\
&\quad +\sqrt{6}r''dz_\eta,\\
&\mathrm{Am}(\gamma\eta')\\
&= 2(d+f)x_{\eta'} + \sqrt{2}\left[(1-s_p)d - 2fR(1-s_v)\right]y_{\eta'}\\
&\quad +\sqrt{6}r''dz_{\eta'},\\
&\mathrm{Am}(\gamma\pi) = 6f.
\end{aligned}
\tag{10}
$$

It should be noted that the amplitude for $J/\psi \to \gamma\eta\,(\eta')$ in (10) is different from that given in [12]. In the $\mathrm{Am}(J/\psi \to \gamma\eta\;(\eta'))$ given in [12], the coefficient of x_η $(x_{\eta'})$, $\frac{2}{\sqrt{6}}\left(d_0 + \frac{f_0}{3}\right)$, should be corrected to become $\frac{2}{\sqrt{6}}\left(d_0 + \frac{f_0}{6}\right)$.

3 Fit results

In the procedure of the fit to experimental data as collected in Tables 1 and 2, we take $x_\omega = y_\phi = 0.998$ and $y_\omega = -x_\phi = -0.059$ [18,22], and the masses of the mesons used in the present work are taken from the new edition of the PDG compilation [19] except for $M_{\eta''} = 1.416\,\mathrm{GeV}$ [6]. The values of the 14 free parameters are determined as follows:

$$
\begin{aligned}
\theta_1 &= 114.2^\circ \pm 6.3^\circ, \quad \theta_2 = 152.1^\circ \pm 2.9^\circ,\\
\theta_3 &= -78.5^\circ \pm 4.6^\circ, \quad g = 1.100 \pm 0.043,\\
e &= 0.124 \pm 0.006, \quad s_v = 0.270 \pm 0.062,\\
s_p &= 0.220 \pm 0.016, \quad r = -0.320 \pm 0.012,\\
r' &= -0.490 \pm 0.046, \quad r'' = -6.530 \pm 0.824,\\
\theta_e &= 78.5^\circ \pm 8.4^\circ, \quad d = -0.143 \pm 0.015,\\
f &= 0.017 \pm 0.003, \quad R = 0.622 \pm 0.023,
\end{aligned}
\tag{11}
$$

with χ^2/d.o.f (χ^2per degree of freedom) $= 10.5/7 = 1.5$. The quarkonia-glueball content of η, η' and η'' is given by

$$
\begin{aligned}
x_\eta &= +0.698 \pm 0.084, \quad y_\eta = -0.690 \pm 0.095,\\
z_\eta &= -0.192 \pm 0.050,\\
x_{\eta'} &= -0.573 \pm 0.099, \quad y_{\eta'} = -0.699 \pm 0.130,\\
z_{\eta'} &= +0.428 \pm 0.046,\\
x_{\eta''} &= -0.430 \pm 0.045, \quad y_{\eta''} = -0.189 \pm 0.039,\\
z_{\eta''} &= -0.883 \pm 0.024.
\end{aligned}
\tag{12}
$$

Based on the above parameters, the predicted results of the decay modes used in our work are presented in Tables 1 and 2.

From the effective lagrangians shown in (4) and (6), one can have

$$
\begin{aligned}
\frac{\Gamma(\eta'' \to \gamma\gamma)}{\Gamma(\pi \to \gamma\gamma)} &= \frac{1}{9}\left(\frac{m_{\eta''}}{m_\pi}\right)^3 (5x_{\eta''} + \sqrt{2}Ry_{\eta''})^2,\\
\frac{\Gamma(\eta'' \to \rho\gamma)}{\Gamma(\omega \to \pi\gamma)} &= 3\left[\frac{(m_{\eta''}^2 - m_\rho^2)m_\omega}{(m_\omega^2 - m_\pi^2)m_{\eta''}}\right]^3 \frac{x_{\eta''}^2}{x_\omega^2},\\
\frac{\Gamma(\eta'' \to \omega\gamma)}{\Gamma(\omega \to \pi\gamma)} &= \frac{1}{3}\left[\frac{(m_{\eta''}^2 - m_\omega^2)m_\omega}{(m_\omega^2 - m_\pi^2)m_{\eta''}}\right]^3\\
&\quad \times \frac{(x_\omega x_{\eta''} - 2y_\omega Ry_{\eta''})^2}{x_\omega^2},\\
\frac{\Gamma(\eta'' \to \phi\gamma)}{\Gamma(\omega \to \pi\gamma)} &= \frac{1}{3}\left[\frac{(m_{\eta''}^2 - m_\phi^2)m_\omega}{(m_\omega^2 - m_\pi^2)m_{\eta''}}\right]^3\\
&\quad \times \frac{(x_\phi x_{\eta''} - 2y_\phi Ry_{\eta''})^2}{x_\omega^2}.
\end{aligned}
\tag{13}
$$

Also, from (8) and (9), one can have the following amplitudes for $J/\psi \to P\;(\gamma)\eta''$:

$$
\begin{aligned}
&\mathrm{Am}(\omega\eta'')\\
&= x_\omega\Big\{(g+e)x_{\eta''} + \sqrt{2}rg\left[\sqrt{2}x_{\eta''} + (1-s_p)y_{\eta''}\right]\\
&\quad +\sqrt{2}r'gz_{\eta''}\\
&+ y_\omega\Big\{\left[g(1-s_v) - 2eR\right]y_{\eta''}\\
&\quad +rg(1-s_v)\left[\sqrt{2}x_{\eta''} + (1-s_p)y_{\eta''}\right]\Big\}\\
&+ y_\omega r'g(1-s_v)z_{\eta''},\\
&\mathrm{Am}(\phi\eta'')\\
&= x_\phi\Big\{(g+e)x_{\eta''} + \sqrt{2}rg\left[\sqrt{2}x_{\eta''} + (1-s_p)y_{\eta''}\right]\\
&\quad +\sqrt{2}r'gz_{\eta''}\Big\}\\
&+ y_\phi\Big\{\left[g(1-s_v) - 2eR\right]y_{\eta''}\\
&\quad +rg(1-s_v)\left[\sqrt{2}x_{\eta''} + (1-s_p)y_{\eta''}\right]\Big\}\\
&+ y_\phi r'g(1-s_v)z_{\eta''},\\
&\mathrm{Am}(\rho\eta'')\\
&= 3ex_{\eta''},
\end{aligned}
$$

Table 1. The predicted and measured width rates for $P \to \gamma\gamma$, $V \to \gamma P$ and $P \to \gamma V$

Decay mode	PDG [19]	Fit	Decay mode	PDG [19]	Fit
$\frac{\Gamma(\eta\to\gamma\gamma)}{\Gamma(\pi\to\gamma\gamma)}$	59.95 ± 7.07	61.50 ± 18.31	$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi\to\gamma\gamma)}$	553.28 ± 69.58	480.69 ± 137.75
$\frac{\Gamma(\rho\to\gamma\eta)}{\Gamma(\omega\to\gamma\pi)}$	0.050 ± 0.019	0.061 ± 0.015	$\frac{\Gamma(\omega\to\gamma\eta)}{\Gamma(\omega\to\gamma\pi)}$	0.0076 ± 0.0013	0.0068 ± 0.0018
$\frac{\Gamma(\phi\to\gamma\eta)}{\Gamma(\omega\to\gamma\pi)}$	0.081 ± 0.006	0.079 ± 0.021	$\frac{\Gamma(\phi\to\gamma\eta')}{\Gamma(\omega\to\gamma\pi)}$	0.00042 ± 0.00022	0.0003 ± 0.00012
$\frac{\Gamma(\eta'\to\gamma\rho)}{\Gamma(\omega\to\gamma\ pi)}$	0.083 ± 0.009	0.088 ± 0.030	$\frac{\Gamma(\eta'\to\gamma\omega)}{\Gamma(\omega\to\gamma\pi)}$	0.0085 ± 0.0012	0.0096 ± 0.0030

Table 2. The predicted and measured branching ratios for $J/\psi \to PV$ and $J/\psi \to \gamma P$

Decay mode	PDG [19](10^{-3})	Fit (10^{-3})	Decay mode	PDG [19](10^{-3})	Fit (10^{-3})
$\rho\pi$	12.7 ± 0.9	11.68 ± 0.69	$K^+\overline{K}^{*-}$ + c.c.	5.0 ± 0.4	5.17 ± 0.33
$K^0\overline{K}^{*0}$ + c.c.	4.2 ± 0.4	4.48 ± 0.33	$\omega\eta$	1.58 ± 0.16	1.50 ± 0.19
$\omega\eta'$	0.167 ± 0.025	0.169 ± 0.070	$\phi\eta$	0.65 ± 0.07	0.67 ± 0.15
$\phi\eta'$	0.33 ± 0.04	0.26 ± 0.09	$\rho\eta$	0.193 ± 0.023	0.184 ± 0.048
$\rho\eta'$	0.105 ± 0.018	0.096 ± 0.034	$\omega\pi$	0.42 ± 0.06	0.42 ± 0.04
$\phi\pi$	< 0.0068	0.0013 ± 0.0001	$\gamma\eta$	0.86 ± 0.08	0.81 ± 0.47
$\gamma\eta'$	4.31 ± 0.03	4.28 ± 1.46	$\gamma\pi$	0.039 ± 0.013	0.039 ± 0.014

Table 3. The predicted results for $\eta'' \to \gamma\gamma$, $\eta'' \to \gamma P$

Decay mode	Fit	Decay mode	Fit
$\frac{\Gamma(\eta''\to\gamma\gamma)}{\Gamma(\pi\to\gamma\gamma)}$	687.89 ± 129.37	$\frac{\Gamma(\eta''\to\gamma\rho)}{\Gamma(\omega\to\gamma\pi)}$	1.27 ± 0.25
$\frac{\Gamma(\eta''\to\gamma\omega)}{\Gamma(\omega\to\gamma\pi)}$	0.14 ± 0.03	$\frac{\Gamma(\eta''\to\gamma\phi)}{\Gamma(\omega\to\gamma\pi)}$	0.01 ± 0.005

Table 4. The predicted results for $J/\psi \to \eta'' + V$ and $J/\psi \to \gamma\eta''$

Decay mode	Fit (10^{-3})	Decay mode	Fit (10^{-3})
$\omega\eta''$	0.36 ± 0.07	$\phi\eta''$	0.16 ± 0.03
$\rho\eta''$	0.03 ± 0.007	$\gamma\eta''$	6.50 ± 2.27

$$
\begin{aligned}
&\mathrm{Am}(\gamma\eta'') \\
&= 2(d+f)x_{\eta''} + \sqrt{2}\left[(1-s_p)d - 2fR(1-s_v)\right] y_{\eta''} \\
&+ \sqrt{6}r''dz_{\eta''}.
\end{aligned} \tag{14}
$$

From these theoretical expressions and the above parameters determined from our fit, one can have the predicted results of the decays involving η'' as shown in Tables 3 and 4, which can provide a consistency test of our results.

4 Discussion

After we established the quark and gluon content of η, η' and η'', it remains to compare our results with others obtained in different contexts. Our results indicate that the dominant component of the physical meson η'' is a glueball, which in agreement with the conclusions given by [7, 9–13] except [24] which concluded that η' is mainly a glueball state. As far as the interpretations of η and η' is concerned, there is a qualitative agreement between our results and those derived from [9–11, 13] that the physical meson η' contains a substantial glueball component, while the physical meson η has a rather small glueball component, however, our results differ from the conclusion given by [25] that η' has a negligible glueball content, also differ from the results given by [12] that η and η' have the nearly equal magnitude of glueball component.

Comparing Table 1 with Table 3, we find that the width of $\eta'' \to \gamma\gamma$ is not smaller than that of η (η') $\to \gamma\gamma$, although the dominant component of η'' is a glueball. It arises from a much larger phase-factor of η'', and also from the fact that η'' mixes with a substantial $u\bar{u} + d\bar{d}$ component with the same sign of the $s\bar{s}$ component. Our results show that a physical meson whose dominant component is a glueball also may have a large partial width of it decaying into two photons, and that it may be dangerous to determine whether a physical state is a glueball by means of the naive prediction that the glueball state should have a rather small partial width of it decaying into two photons. Our results also indicate that the mixing of glueball and quarkonia can significantly change the prediction on the properties of the pure glueball.

From Tables 2 and 4, the relation of $\mathrm{Br}(J/\psi \to \gamma\eta)$: $\mathrm{Br}(J/\psi \to \gamma\eta')$: $\mathrm{Br}(J/\psi \to \gamma\eta'') = 1.0 : 5.2 \pm 3.5 : 8.0 \pm 5.4$ is obtained, which is in agreement with the prediction given by [9] that $\mathrm{Br}(J/\psi \to \gamma\eta)$: $\mathrm{Br}(J/\psi \to \gamma\eta')$: $\mathrm{Br}(J/\psi \to \gamma\eta'') = 1.0 : 5.0 : 6.8$. With $z_{\eta''} > z_{\eta'} > z_\eta$, the relation of $\mathrm{Br}(J/\psi \to \gamma\eta'') > \mathrm{Br}(J/\psi \to \gamma\eta') > \mathrm{Br}(J/\psi \to \gamma\eta)$ favors the assumption that the J/ψ

radiative decays go mainly through the gluonic component [11]. Also, the value of the parameter r'' ($r'' > 1$, $r'' \neq r'/\sqrt{3}r$), on the one hand, indicates that the effective coupling strength of the processes shown in Fig. 2(ii) is larger than that of Fig. 2(i), which also favors the assumption that the J/ψ radiative decays go mainly through the gluonic component [11], on the other hand, indicates that the coupling strength of Fig. 2(ii) relative to that of Fig. 2(i) is different from the coupling strength of Fig. 1(iv) relative to that of Fig. 1(iii), which suggests that the non-perturbative effects in the processes as shown in Fig. 1(iii), (iv) and Fig. 2(i), (ii) are rather large.

From our results, $\Gamma(\eta'' \to \rho\gamma)/\Gamma(\eta'' \to \omega\gamma)$ is predicted to have the value of 9.07 ± 2.6, which is in excellent agreement with the prediction given by [26,27] that $\Gamma(\eta'' \to \omega\gamma) = \frac{1}{9}\Gamma(\eta'' \to \rho\gamma)$, and is also consistent with the prediction given by [28,29] that $\Gamma(\eta'' \to \rho\gamma) > \Gamma(\eta'' \to \omega\gamma)$. Also, in our present analysis, R has been left as a free parameter to fit. R is determined to be the value of 0.622 ± 0.023, which is consistent with the value of 0.642 as adopted by [30].

Finally, we want to note that if we set $x_\omega = 1$, $y_\omega = 0$, $x_\phi = 0$ and $y_\phi = 1$, i.e., the effects of non-ideal ω–ϕ mixing are neglected, we find that the unknown parameters are not much changed, but χ^2 increases to 19.43, which suggests that effects of non-ideal ω–ϕ mixing on the quarkonia-glueball content of η, η', η'' are very small, however, as pointed out by Bramon et al. [22,31], the introduction of the small, but certainly non-vanishing, departure of ω and ϕ from the ideally mixed states $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ can improve the description on the amplitudes of the decay processes used in this present work.

5 Summary and conclusion

Considering corrections arising from SU(3) flavor breaking, non-ideal ω–ϕ mixing, DOZI and electromagnetic amplitudes of $J/\psi \to PV$, and three-gluon-annihilation amplitudes of $J/\psi \to \gamma P$, we improve the description of the decay amplitudes of $P \to \gamma\gamma$, $V \to \gamma P$, $P \to \gamma V$, $J/\psi \to PV$, and $J/\psi \to \gamma P$. The quarkonia-glueball structure of η, η' and $\eta(1410)$ is deduced from the experimental data available at present. We find that η (η') contains about $(48.7 \pm 8.3)\%$, $(47.6 \pm 9.2)\%$, $(3.7 \pm 1.3)\%$ ($(32.9 \pm 7.8)\%$, $(48.8 \pm 12.8)\%$, $(18.3 \pm 2.8)\%$) non-strange quarkonium, strange quarkonium and glueball components, respectively, and that $\eta(1410)$ is mainly a glueball state (about $(78 \pm 3.0)\%$ glueball component), mixed with $(18.5 \pm 2.6)\%$ non-strange quarkonium and $(3.5 \pm 1.0)\%$ strange quarkonium. Also, we predict that $\Gamma(\eta(1410) \to \gamma\gamma)/\Gamma(\pi \to \gamma\gamma) = 687.89 \pm 129.37$, $\Gamma(\eta(1410) \to \gamma\rho)/\Gamma(\omega \to \gamma\pi) = 1.27 \pm 0.25$, $\Gamma(\eta(1410) \to \gamma\omega)/\Gamma(\omega \to \gamma\pi) = 0.14 \pm 0.03$, $\Gamma(\eta(1410) \to \gamma\phi)/\Gamma(\omega \to \gamma\pi) = 0.01 \pm 0.005$, $\mathrm{Br}(J/\psi \to \omega\eta(1410)) = (0.36 \pm 0.07) \times 10^{-3}$, $\mathrm{Br}(J/\psi \to \phi\eta(1410)) = (0.16 \pm 0.03) \times 10^{-3}$, $\mathrm{Br}(J/\psi \to \rho\eta(1410)) = (0.03 \pm 0.007) \times 10^{-3}$, and $\mathrm{Br}(J/\psi \to \gamma\eta(1410)) = (6.50 \pm 2.27) \times 10^{-3}$. The expected $J/\psi \to \gamma\eta(1410)$ branching ratio is large and should be observable at BEPC/BES with 50 million J/ψ events in the near future.

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